

**Estimation of the van Genuchten-Mualem parameter  $\alpha$   
and the saturated hydraulic conductivity from SIP measurements**

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The application of hydrological methods to determine soil hydraulic properties can be time-consuming and may become more efficient when supported by geophysical methods. Due to the dependence of the complex electrical resistivity on the pore fluid, the porosity, the size of pores and grains, respectively, and the interface between the pore fluid and the matrix, SIP might be suitable to support the hydrological investigations (e.g. Hördt et al. 2009; Breede et al. 2011). To identify relationships between complex geoelectrical parameters derived from SIP spectra and soil hydraulic properties, we performed SIP and hydrological investigations in the laboratory on the same samples, containing different unconsolidated sediments, covering a wide range of texture classes. The so-called van Genuchten-Mualem model, which was suggested by van Genuchten (1980) based on a model by Mualem (1976), is a widespread approach in soil physics to describe the hydraulic conductivity:

$$K(S_e) = K_s S_e^l \left( 1 - \left( 1 - S_e^{1/m} \right)^m \right)^2$$

as a function of the effective saturation:

$$S_e(h) = \left( 1 + \alpha |h|^n \right)^{-m}$$

with the pressure head  $h$ , the saturated hydraulic conductivity  $K_s$  and the empirical parameters  $\alpha$ ,  $l$ ,  $n$ , and  $m = 1 - 1/n$ . We apply this model to interpret the results of hydrological laboratory investigations, which we performed using the multi-step outflow method and the evaporation method with the HYPROP<sup>©</sup> device (UMS GmbH, Munich), respectively. Detailed descriptions of these hydrological methods can be found e.g. in Breede et al. (2011) and Peters and Durner (2008).

For the SIP measurements, we studied the frequency range between 10 mHz and 100 Hz with a VMP 3 impedance spectrometer (Princeton Applied Research). The SIP spectra were evaluated with the Debye decomposition approach, which was suggested by Nordsiek and Weller (2008) to derive characteristic parameters like the mean relaxation time  $\tau_{mean}$ , the normalized total chargeability  $m_n$ , and the non-uniformity parameter  $U_\tau$  from the measured SIP spectra.

We considered eight samples of unconsolidated sediments. The set of investigated material covers a variety of different grain sizes. It includes fine-textured soil samples (GGL, WFB, VRD) as well as sandy soil samples (RBE, RBC, STO, SSU) and a sample of pure medium and coarse sand (LAB). All samples were saturated with calcium chloride solutions of different ionic strengths, ranging from 0.001 to 0.02 mol per litre.

We merged the results of SIP measurements at different ionic strengths of the saturating fluid and compared the SIP parameters with the hydrological van Genuchten-Mualem parameters. We found a strong correlation with a correlation coefficient of  $R = -0.76$  between  $m_n$  and  $\alpha$  from the van Genuchten-Mualem model. After reconsideration of the measured data and exclusion of two samples that might have been affected by equilibration processes after saturation, we formulated the empirical relationship:

$$\alpha_{SIP} = 1.69 \cdot 10^{-2} \cdot m_n^{-0.95}$$

with  $\alpha_{SIP}$  in  $\text{cm}^{-1}$  and  $m_n$  in  $\text{mS m}^{-1}$ . The correlation coefficient is  $R = 0.91$  and the adjusted coefficient of determination is  $R_{adj}^2 = 0.81$ . In Fig. 1 (left), we show the estimated parameter  $\alpha_{SIP}$

and the parameter  $\alpha_{MSO}$  resulting from the hydrological investigations.

Another relationship was observed for the saturated hydraulic conductivity and the normalized chargeability. Here, we found the power law:

$$K_s \text{ SIP} = 6.35 \cdot 10^{-11} m_n^{-1.60} \rho_0^{-1} \sigma_w^{-1}$$

with  $R = 0.87$  and  $R_{adj}^2 = 0.74$  for the estimation of the saturated hydraulic conductivity  $K_s \text{ SIP}$  (in  $\text{m s}^{-1}$ ) from the normalized chargeability  $m_n$  (in  $\text{S m}^{-1}$ ), the DC resistivity  $\rho_0$  (in  $\Omega \text{ m}$ ), and the conductivity of the saturating fluid  $\sigma_w$  (in  $\text{S m}^{-1}$ ). In Fig. 1 (right), we display the estimated hydraulic conductivity and the measured hydraulic conductivity  $K_s \text{ MSO}$ .

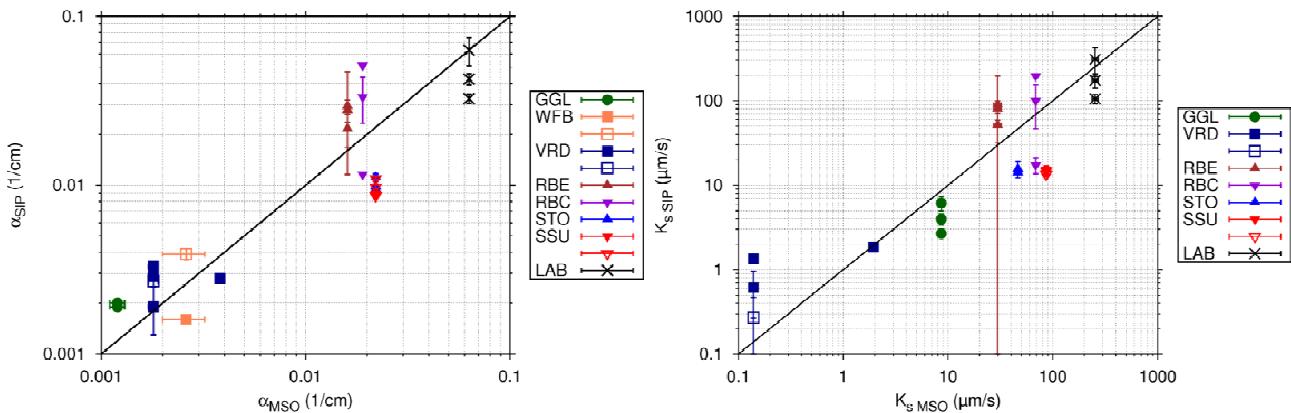


Fig. 1: Estimation of the van Genuchten-Mualem parameter  $\alpha$  (left) and estimation of the saturated hydraulic conductivity (right) with parameters resulting from SIP measurements. The lines indicate identity between the measured values and the estimated values (Nordsiek et al. subm.).

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