Anisotropic complex conductivity inversion

J. Kenkel(1) and A. Kemna(1)

(1) *Department of Geodynamics and Geophysics, University of Bonn, Germany*

Introduction

In spectral induced polarization, impedance measurements at multiple frequencies are used to derive the subsurface spatial distribution of complex electrical conductivity by means of an inversion algorithm. Presently, these algorithms assume an isotropic complex electrical conductivity. However, as has been shown already for DC conductivity inversion, the assumption of isotropy can yield misleading or unnecessarily complex results (e.g. Nguyen et al. 2007). We propose a new inversion approach supporting anisotropic complex electrical conductivity distributions. The algorithm is based on CRTomo (Kemna 2000), an established code for isotropic complex electrical conductivity inversion. We present a synthetic study which highlights the improvements and limitations of an anisotropic complex conductivity inversion approach.

Inversion algorithm

The 2-D algorithm is based on 2.5-D finite-element forward modelling (Kenkel et al. 2012), and anisotropic complex conductivities are considered in the computation of the predicted impedances, the sensitivities, and the iterative model update in the Gauss-Newton inversion scheme.

We assume a diagonal complex conductivity tensor in terms of Cartesian coordinates. Correspondingly, the sensitivities are defined as derivatives of the complex electric potential, ϕ, with respect to the electrical conductivity values in the different directions, σ_i, as:

$$\frac{\partial\phi}{\partial\sigma_x}, \frac{\partial\phi}{\partial\sigma_y}, \frac{\partial\phi}{\partial\sigma_z}.$$

This allows the representation of anisotropy caused by horizontal or vertical structures (e.g. layering).

The iterative model update is carried out similar to the approach by Kemna and Binley (1996), but with extended equations accounting for the complex conductivity tensor. For regularization, a smoothness constraint is imposed for each component of the complex conductivity tensor. In addition, anisotropy is penalized, i.e., the most isotropic model possible is sought, by implementing an anisotropy penalty function which minimizes the term:

$$\begin{pmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \end{pmatrix}^{\mathrm{T}} \begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix} \begin{pmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \end{pmatrix},$$

for each model cell, following the approach of Pain et al. (2003).

Synthetic modelling example

We conducted a synthetic study where we imaged a perpendicular fault model from surface measurements. The original model and the inversion result are displayed in Fig. 1 as sections of the magnitude of the complex resistivity in *x* and *y* directions. It features an anisotropic fault region with resistivity values in *x* and *z* directions of 10 and 1 Ωm, respectively, embedded in an isotropic background of 1 Ωm. A dipole-dipole measurement configuration employing 40 surface electrodes is assumed. The anisotropic inversion algorithm is capable of correctly resolving the anisotropic fault region. However, the resistivity is slightly underestimated in *x* direction and overestimated in *z* direction. Nevertheless, the successful reconstruction of the shape and position of the anisotropic fault region in a relatively isotropic background seems promising.

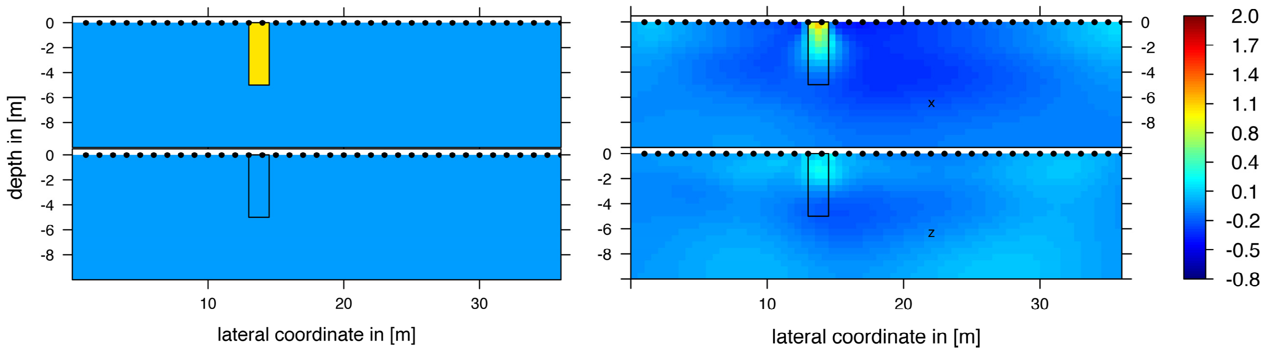

Fig. 1: Original (left) and reconstructed (right) resistivity in log10(Ω m) in x and y directions (top and bottom, respectively) featuring an isotropic fault region in an isotropic background (1 Ω m). The fault region is characterised by resistivities of 10 Ω m and 1 Ω m in x and z directions, respectively. The black dots denote the electrode positions.

Conclusions

We think that accounting for anisotropy in complex conductivity inversion should improve the interpretation of resistivity/IP imaging data in many situations. We successfully verified our implementation with synthetic examples targeting the detectability of anisotropy. As a false test, we successfully inverted a synthetic example with isotropic conductivities. First results of the synthetic study indicate that the algorithm is able to resolve anisotropic conductivities while underestimating the original anisotropy factor.

We acknowledge support by the SFB/TR 32 "Patterns in soil-vegetation-atmosphere systems: monitoring, modelling and data assimilation", funded by the German Science Foundation (DFG).

References

Kemna, A. and Binley, A., 1996. Complex electrical resistivity tomography for contaminant plume delineation. *Proc. 2nd meeting Environ. Eng. Geophys. Soc.*, 2-5 September 1996, Nantes, 196-199.

Kemna, A., 2000. Tomographic inversion of complex resistivity - theory and application. Ph.D. thesis, Ruhr-Universität Bochum.

Kenkel, A., Hördt, A. and Kemna, A., 2012. 2D modelling of induced polarization data with anisotropic complex conductivities. *Near Surface Geophys.*, 10, 533-544.

Pain, C.C., Herwanger, J.V., Saunders, J.H., Worthington, M.H. and de Oliveira, C.E., 2003. Anisotropic resistivity inversion. *Inverse Probl.*, 19, 1081-1111.

Nguyen, F., Garambois, S., Chardon, D., Hermitte, D., Bellier, O. and Jongmans, D., 2007. Subsurface electrical imaging of anisotropic formations affected by a slow active reverse fault, Provence, France. *J. Appl. Geophys.*, 62, 338-353.