Spectral induced polarization: frequency domain versus time domain

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Last decades, there has been a growing interest in the use of the Spectral Induced Polarization method in hydrogeology, contaminant hydrology, biogeophysics and mining geophysics. Field works are frequently carried out in a so-called Time Domain (TD), whereas the physics, which governs the IP phenomenon, is studied in lab conditions on the basis of Frequency Domain (FD) measurements. In FD, the complex dielectric constant, electrical conductivity (or its inverse, the resistivity) is measured in a wide frequency range (over 9 decades, from 1 mHz to 1 MHz, in laboratory conditions, and over 3 decades, from 0.1 to 100 Hz, in field conditions). For the resistivity, measurements are based on the Ohm's Law,

$$U(f)=\frac{1}{K}\rho(f)I(f), \tag{1}$$

where $U(f)$, $I(f)$ and $\rho(f)$, are the voltage, current and resistivity, respectively, f is the frequency and K is the geometrical factor.

In TD, voltage decays after cut-off of the electrical current are measured. The current waveform is a sequence of pulses of opposite polarity and pauses of the same duration. Decays after pulse sequences of different duration are obtained to cover three to six decades of time. Measurements are again based on the Ohm's Law, which is presented in the form of convolution,

$$U(t)=\frac{1}{K}\int_0^\infty \rho(t_0)I(t-t_0)\,dt_0\,, \tag{2}$$

where t is the time after switching the DC on, and t_0 is the integration variable.

From theoretical point of view, FD data are easier to interpret just because the product in Eq. (1) is simpler than the convolution in Eq. (2). Moreover the phase IP spectra frequently contain maxima, which is easier to analyze, comparing monotonous IP decays. However TD measurements are an attractive alternative because they are less time consuming than FD measurements. Table 1 presents IP parameters in TD and FD.

To date there is no universal link between the parameters measured in TD and FD. This link can be obtained on the basis of data fitting to empirical models (e.g., the Cole-Cole model). Then, on the basis of the model parameters, IP responses in FD or TD can be calculated. However these models are not universal.

We suggest using a Debye decomposition approach to link spectral data obtained in TD and FD. The Debye decomposition allows deconvolution of an IP signal to a sum of elementary polarizing responses describing by the Debye relaxation model and with different relaxation times. Distribution of magnitude of the Debye responses as a function of the relaxation time is obtained with DD. Although this approach is phenomenological it is universal because it can be applied to any type of data in FD and TD.

We applied the Debye decomposition approach to three datasets obtained in both TD and FD. These datasets are: (1) synthetic data calculated for an RC circuit shown in insert to Fig. 1b; (2) experimental data obtained on the same circuit; and (3) data measured on a mixture of calibrated commercial sand ($r = 0.2\ldots0.3$ mm) and 10 % of commercial lead shot ($r = 2.2$ mm) saturated with NaCl solution ($\rho = 70$ Ωm). Figure 1 shows the respective relaxation time distributions.

For the synthetic test (Fig. 1a) the RTDs obtained from FD and TD are very close, for the experimental test (Fig. 1b) the difference between them slightly increases, and for the experiment with lead shot the difference increases again. However for these three datasets the agreement between two types of RTD is reasonable. Only for the lead shot experiment the peak relaxation time values obtained from FD and TD data differ by a factor of about 2.5. This difference between RTD obtained on the basis of FD and TD data occurs because the Debye decomposition approach resolves an ill-posed problem. However in the forward modelling both type of RTD allow robust calculation of TD and FD data. We suggest therefore the Debye decomposition approach as the link between FD and TD data. Based on RTD, FD data can be easily transformed to TD data and vice versa.

Parameter		Type of parameter
TD	FD	
Polarizability $\eta(t)=U(t)/U_0$, $U(t)$ and U_0 are the off-time and on-time voltages, respectively	Phase $\varphi=\arctan\left({}^{\rho''}\!/\!{}_{\rho'}\right)$, ρ' and ρ'' are the real and imaginary components of the resistivity, respectively	Local
Chargeability $m=\frac{1}{t_{\max}-t_{\min}}\int_{t\min}^{t\max}\eta(t)\,dt$, $\lvert t_{\max}-t_{\min}\rvert$ is the integration time	Frequency effect $FE=\frac{\rho_{lf}-\rho_{hf}}{\rho_{lf}}$, ρ_{lf} and ρ_{hf} are the resistivity values at low and high frequencies, respectively	Partial
Normalized chargeability $m_n={}^{m}\!/\!{}_{\rho}\sim\sigma''$	Metal Factor $MF={}^{FE}\!/\!{}_{\rho}\sim\sigma''$, σ'' is the imaginary component of the conductivity	Normalized partial
Total chargeability, obtained numerically from the Debye Decomposition approach	Total chargeability $M=\frac{\rho_0-\rho_\infty}{\rho_0}$, ρ_0 and ρ_∞ are the low and high frequency asymptotes of the resistivity, respectively	Global
Normalized total chargeability	Normalized total chargeability $M_n=\frac{\rho_0-\rho_\infty}{\rho_0^2}$	Normalized global
Polarizability vs. time	Phase vs. frequency	Spectral

Table 1. IP parameters in FD and TD.

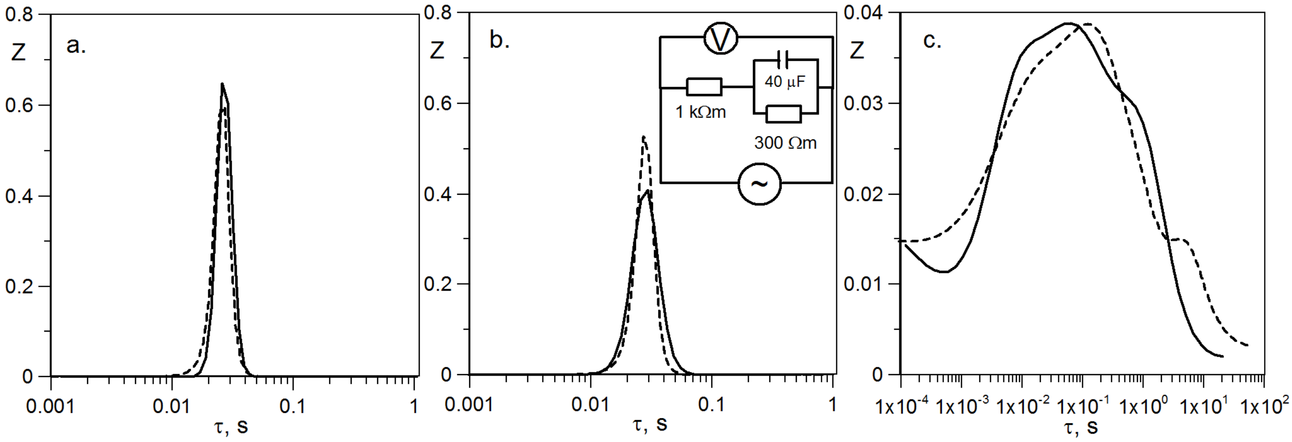

Fig. 1: Relaxation time distributions obtained for synthetic (a) and experimental (b) RC-tests and the lead shot experiment (c). Solid and dashed lines are RTDs obtained from FD and TD data, respectively.